Chromatic dispersion measurement in optical fibers

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Chromatic dispersion (CD) is the phenomenon leading to delay differences between different spectral components when propagating through an optical fiber. This fact may result in a loss of information effective capacity of the fiber. Thus, CD becomes a notable consideration and must be taken into account when developing fiber optic equipment for use in telecommunications. The aim of this project is to characterize CD via four different experiments, as well as discussing their advantages and limitations.

I. THEORETICAL BACKGROUND

Dispersive media add a phase as a function of the wavelength that is travelling. It is modeled as a transfer function with the form $e^{-j\beta L}$, where L is the length of the fiber [1]. The signal has a center frequency w_0 with small variations, so $\beta(\omega)$ may be expressed with its Taylor expansion:

$$\beta(\omega) = \beta(\omega_0) + \frac{\partial \beta}{\partial \omega}\Big|_{\omega_0}(\omega-\omega_0) + \frac{1}{2}\frac{\partial^2 \beta}{\partial \omega^2}\Big|_{\omega_o}(\omega-\omega_0)^2 + \cdots \quad (1)$$

Let $\beta_0 = \beta(w_0)$ and $\beta_i = \frac{\partial^i \beta}{\partial w^i}\Big|_{\omega_0}$, which leads to:

$$\beta(\omega) = \beta(\omega_0) + \beta_1(\omega-\omega_0) + \frac{1}{2}\beta_2(\omega-\omega_0)^2 \quad (2)$$

The concepts of phase and group velocity are related with the first and second terms of the expansion as $v_{ph} = \frac{w_0}{\beta_0}$ and $v_g = \frac{1}{\beta_1}$, while the third term contains information about dispersive effects.

τ_g is defined as the time the signal needs to travel a distance L, and so we have:

$$v_g = \frac{L}{\tau_g} \Rightarrow \tau_g = \beta_1 L \quad (3)$$

β_2 can be expressed as:

$$\beta_2 = \frac{1}{L}\frac{\partial(\beta_1 L)}{\partial \omega}\Big|_{\omega_0} = \frac{1}{L}\frac{\partial \tau_g}{\partial \omega}\Big|_{\omega_0} \quad (4)$$

The dispersion coefficient determines how the group delay varies with λ, so it is defined as following:

$$D = \frac{1}{L}\frac{\partial \tau_g}{\partial \lambda}\Big|_{\lambda_0} = \frac{\partial \omega}{\partial \lambda}\Big|_{\omega_0}\frac{1}{L}\frac{\partial \tau_g}{\partial \omega}\Big|_{\omega_0} = -\frac{2\pi c}{\lambda_0^2}\beta_2 \quad (5)$$

II. EXPERIMENTAL RESULTS

The following experiments are made to characterize the dispersion coefficient of an optical fiber. Each experiment uses the same diagram.

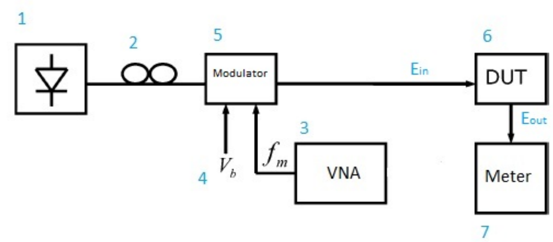

FIG. 1 Basic structure of the experiments done.

A signal with the carrier frequency is emitted with the Tunable Laser (1 in FIG. 1) and introduced into a modulator. Before arriving, it encounters a polarization control (2 in FIG. 1) which changes the polarization for the modulator.Thus, the polarization control must be set in such a way that the power received is maximum before starting the experiment.

The modulator (5 in FIG. 1) will be either a Push-Pull or a Dual-Electrode, both explained later on. The modulator has another input which corresponds to the optical envelope (3 in FIG 1), consisting on either a pulse or a periodic signal with a frequency in the RF range. It is generated by a pulse generator or by the Vectorial Network Analyser (VNA). At that point the signal can be measured with a photodetector followed by a meter (either an oscilloscope or a VNA) (7 in FIG 1). Results with (6 in FIG. 1) and without a dispersive element between the modulator and the photodetector will be compared in order to characterize dispersion.

MZM PARAMETERS

In all of our experiments, a Mach-Zehnder modulator is used. It consists of an input waveguide which is split into two interferometer arms. In a push-pull configuration, when the same voltage, with different sign, is applied to the arms, the phase acquired by the beam is different in each of those. Thus, when the arms are recombined, the phase difference between them results in an amplitude modulation.

The transfer function for the Mach-Zehnder modulator is plotted in FIG. 1.

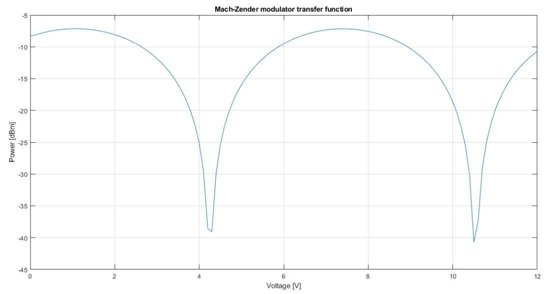

FIG. 2 Mach-Zehnder modulator transfer function

From it, some parameters can be induced. V_π, defined as the voltage that has to be applied in order to go from a minimum to a maximum of the transfer function, is found to be, approximately, $V_\pi = 3V$.

A. Amplitude fading

The field at the output of the MZM is the interference of its two biased branches. Is the one that will be introduced into the dispersive media:

$$E_{in} = \frac{E_0}{2}\left(e^{j(\theta_B+\theta_{RF}\cos(w_{RF}t))} + e^{-j(\theta_B+\theta_{RF}\cos(w_{RF}t))}\right) \tag{6}$$

$$\theta_B = \frac{\pi V_B}{2V_\pi} \tag{7}$$

$$\theta_{RF} = \frac{\pi V_{RF}}{2V_\pi} << 1 \tag{8}$$

Where V_B is the bias voltage and V_{RF} is the amplitude of the RF signal.

This approximation ignores second order terms and leads to a simplest expression of the incident field:

$$E_{in} \simeq \frac{E_0}{2}\left(2\cos\theta_B - 2\theta_{RF}\sin\theta_B\cos\left(w_{RF}t\right)\right) \tag{9}$$

At the output of the fiber, the field has an added phase due to dispersion:

$$\begin{aligned} E_{out} = & E_0\Bigg[e^{-j\beta_0 L}\cos\theta_B + \\ & -\theta_{RF}\sin\theta_B\frac{1}{2}\Big(e^{jw_{RF}t}e^{-j\left(\beta_0+\beta_1 w_{RF}+\frac{\beta_2}{2}w_{RF}^2\right)L} + \\ & + e^{-jw_{RF}t}e^{-j\left(\beta_0-\beta_1 w_{RF}+\frac{\beta_2}{2}w_{RF}^2\right)L}\Big)\Bigg] \end{aligned} \tag{10}$$

The detected photocurrent is the squared modulus of the field.

The VNA will provide the scattering parameter S_{21}, that is the ratio between the amplitudes at its output and input. With the previous expressions found, one can end up that it is proportional to a term containing the dispersion coefficient.

$$|s_{21}| \propto \cos\left(\frac{\beta_2}{2}\omega_{RF}^2 L\right) \tag{11}$$

Which, taking expression (5) into account, can be expressed as

$$|s_{21}| \propto \cos\left(\frac{\pi}{c}D\lambda_0^2 f^2 L\right) \tag{12}$$

One of the experiments carried out to determine dispersion consists on finding the nulls of the photo-detected current at a fixed λ_0 of the laser and varying the modulation frequency f.

For that, a Chromatic Dispersion Compensation (CDC) (or Fiber Bragg Grating (FBG)) fiber of nominal dispersion coefficient value $D = -1252.55\frac{ps}{nm}$ is used.

Fixing $\lambda_0 = 1559.12nm$ and by using VNA, the first two zeros are found at 5.5GHz and at 11.3GHz. Hence, making π the distance between the zeros of the cosine:

$$\frac{\pi}{c}\lambda_0^2 D(11.3^2 - 5.5^3) = \pi$$

$$\Rightarrow D = \frac{c}{\lambda_0^2(11.3^2 - 5.5^3)} \tag{13}$$

Then, the dispersion is found to be $D = 1266.56\frac{ps}{nm}$, which gives a relative error of $\varepsilon = 1.12\%$. Note that with this method, unlike with the others as it will be seen later, the sign of D cannot be determined.

B. Group delay

Another method to achieve dispersion characterization is based on the group delay measurement. In this technique, as opposed to the amplitude fading procedure, the modulation frequency ω_{RF} is kept constant, whereas the wavelength of the tunable laser is what is modified. By doing so, a variation in the RF phase is observed in the VNA set to measure a fixed frequency of 500MHz and, thanks to it, the dispersion coefficient can be computed through the following expressions:

$$\tau_g = -\frac{d\varphi}{d\omega} \tag{14}$$

$$D = \frac{d\tau_g}{d\lambda} \tag{15}$$

Which can be approximated by

$$\Delta\tau_g = -\frac{\Delta\varphi}{\omega_{RF}} \tag{16}$$

$$D = \frac{\Delta\tau_g}{\Delta\lambda} \tag{17}$$

Thus, working at 500MHz and varying the wavelength from 1557.20nm to 1561.30nm, the following graph representing the phase of the received signals in degrees (°) with respect to the wavelength λ in nm is obtained (FIG. 2):

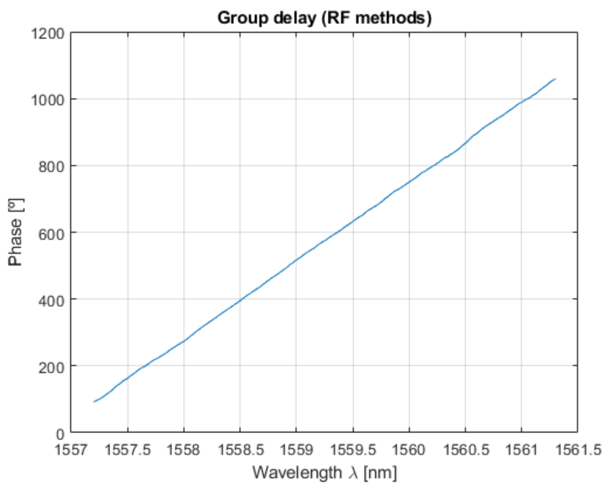

FIG. 3 Representation of the phase with respect of the wavelength at a fixed RF frequency of 500MHz

Therefore, by choosing λ=1577.74nm and λ=1561.30nm, which have a value of the phase φ=218.471° and φ=348.377° respectively, and by using the approximate formulas (16) and (17), the dispersion coefficient D is calculated.

$$\Delta\tau_g = \frac{218.471° - 348.377°}{360° \cdot 500MHz}$$

$$\Delta\lambda = 1561.30nm - 1577.74nm$$

$$\Rightarrow D = \frac{\Delta\tau_g}{\Delta\lambda} \tag{18}$$

Therefore, the value of the dispersion coefficient found is $D = -1288.75\frac{ps}{nm}$. Hence, the relative error by using this method is $\varepsilon = 2.89\%$.

C. Temporal pulse delay

A different procedure consists on working with a pulse generator, and measuring the relative output temporal delay (or group delay) when changing the laser wavelength.

Two values for the wavelength were chosen to be λ_1= 1557.52 nm and λ_2= 1561.00 nm. For those values, relative group delay values of the output pulse with respect to the input one were found to be Δ_1=56.73ns and Δ_2=51.84ns, respectively.

Recalling that dispersion can be obtained as the derivative of the group delay with respect to the wavelength, and considering a linear approximation is valid, the obtained value for dispersion is in this case $D = -1405.17\frac{ps}{nm}$ giving a relative error of $\varepsilon = 12.18\%$.

D. AMBC

In this method, which stands for Asymmetric Mode and Bias Control, in contrast to the other three, a dual-electrode Mach-Zehnder Modulator is used. This one, unlike the Push-Pull MZM, applies bias voltage to only one of the arms. Due to this fact, now the dependence photodetected current is different and it is on the bias voltage that is being applied as it can be seen in the following equations[2]:

$$E_{in} = \frac{E_0}{2}\left[e^{j\theta_B} + e^{j\theta_{RF}\cos(\omega_{RF}t)}\right] \tag{19}$$

Where, $\theta_B = \frac{V_B\pi}{V_\pi}$ and $\theta_{RF} = \frac{V_{RF}\pi}{V_\pi}$. For $\theta_{RF} << 1$, which means $V_{RF} << V_\pi$:

$$E_{in} \simeq \frac{E_0}{2}e^{j\frac{\theta_B}{2}}\left[2\cos\left(\frac{\theta_B}{2}\right) + e^{-j\frac{\theta_B}{2}}j\theta_{RF}\cos(\omega_{RF}t)\right] \tag{20}$$

$$\begin{aligned} E_{out} =& \frac{E_0}{2}e^{j\frac{\theta_B}{2}}e^{-j\beta_0 L}\left[2\cos\left(\frac{\theta_B}{2}\right)+\right. \\ &\left. + j\theta_{RF}e^{-j\frac{\theta_B}{2}}e^{-j\frac{\beta_2}{2}\omega_{RF}^2 L}\cos(\omega_{RF}[t-\beta_1 L])\right] \end{aligned} \tag{21}$$

Therefore, and by normalizing and disregarding quadratic terms, the photodetected current is:

$$\begin{aligned} i_{PD} = 2\cos\left(\frac{\theta_B}{2}\right)\sin\left(\frac{\theta_B}{2} + \frac{\beta_2}{2}\omega_{RF}^2 L\right)\cdot \\ \cdot\cos(\omega_{RF}[t-\beta_1 L]) \end{aligned} \tag{22}$$

Thus, the expression needed in order to compute the dispersion coefficient D comes from the sine in (22).

$$\frac{\theta_{B,1}}{2} - \left(\frac{\theta_{B,2}}{2} - \frac{LD\lambda_0^2}{c}\pi f_{RF}^2\right) = 0 \tag{23}$$

With $\theta_{B,1}$ the value of θ_B at the second zero of the measure without DUT and $\theta_{B,2}$ the value of θ_B at the second zero of the measure with DUT. Substituting the value of θ_B in (23)

$$\Rightarrow D = \frac{c(V_{B,2} - V_{B,1})}{2V_\pi \lambda_0^2 f_{RF}^2} \tag{24}$$

With this being stated, the procedure used in order to have the correct measurements and, consequently, the proper result of the dispersion coefficient D is as follows. Through the control of the devices by the GPIB bus and by using a program with the ability of both communicating with the device and taking the data that it measures, the bias voltage V_B is changed and with a fixed wavelength and RF frequency the power (in dBm) is measured for each value of V_B.

Hence, with λ_0=1559.25nm, f_{RF}=1GHz, and a variation of V_B from 0V to 12V with a step of 0.01V, the following plot representing the power with respect to the bias voltage is obtained (FIG. 3):

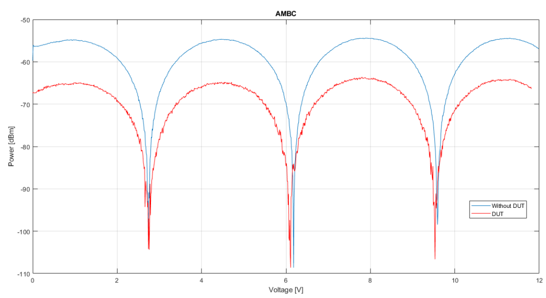

FIG. 4 Power with respect to the bias voltage V_B

As it can be seen, with this values, three zeros are observed. V_π is obtained with the following expression $\frac{V_3 - V_1}{2}$, where V_3 and V_1 are the voltage at the third and first zeros of the transfer function respectively. Once this value is calculated, and by using expression (24), the dispersion coefficient is computed. The values obtained are: $V_1 = 2.74V$, $V_3 = 9.59V$, $V_\pi = 3.425V$, $V_{B,1} = 6.18V$ and $V_{B,2} = 6.11V$. Then, the dispersion coefficient has a value of $D = -1260.95\frac{ps}{nm}$, which means a relative error of $\varepsilon = 0.67\%$.

III. CONCLUSIONS

Four methods have been studied to characterize dispersion. Having in mind that the nominal value of the CDC fiber is $D = -1252.55\frac{ps}{nm}$

Technique	D $[\frac{ps}{nm}]$	ε [%]
Amplitude fading	1266.56	1.12
Group delay	-1288.75	2.89
Temporal pulse delay	-1405.17	12.18
AMBC	-1260.95	0.67

Comparing the results obtained, it is verified that temporal pulse delay is the less precise method used. That is because in practice, the pulse generated had disturbances so it differed from a Gaussian pulse. In addition, the photodetected signal had also disturbances, implying inevitable error in the pulse peak delay measurement.

In the second position we have amplitude fading and group delay. With amplitude fading practical limitations are encountered with the equipment required. As determined frequencies are searched for, all the devices bandwidth can be exhausted without ending with any zero of the photodetected current. Moreover, even if the system achieves the required frequency, the larger it is, the worse is the frequency precision of our dispersion characterization, because the range of frequency to the which a specific value of dispersion is associated increases. Group delay has as a limitation the need of a tunable laser, as at least two values of different wavelengths are needed.

AMBC procedure avoids this limitations. The fact of working with the bias allows to choose the frequency at which it is desired to work, and at the end only the difference between nulls as a function of the bias voltage is needed to determine dispersion. This solves the problem of not achieving a required frequency with the device, or the poor resolution working at high frequencies. Also, single wavelength value is needed, so it a tunable laser is not required.

It can be seen that each experiment relies on different parameters and different magnitudes are measured in each of them with different relative error. Due of this fact, one needs to consider the option of adapting the characterization of dispersion method to the laboratory facilities, precision requirement and available time, among others.

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[1] G. Pecere. *Spectral Amplitude and Phase Characterization of Optical Devices by RF scan*, Master Thesis, UPC, Barcelona, 2010.

[2] M.C.Santos, S.P.De Bernardo-Rodi, M.A Mitre-Gutiérrez *New Modulation Zero-Shift Method to Characterize Fast Group Delay Ripple of Dispersion-Compensating Fiber Bragg Gratings*, IEEE Photonics Technology Letters, 19(17), 2007.